Skew Quadrupole Tuning and Vertical Dispersion in the Tevatron

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1 Introduction

Typical tuning procedures for the Tevaton have included minimizing the global coupling by adjusting skew quadrupole circuits to produce a minimum difference between the "horizontal" and "vertical" betatron tunes. In the early days of the Tevatron this was performed by small adjustments to the main skew quadrupole circuit, T:SQ. Since those times, the Tevatron has become a strongly coupled accelerator primarily due to the relaxation, or creep, of the superconducting coils within the iron yoke of the main dipole magnets which generates a systematic skew quadrupole moment, a_1, around the ring.[1] Additionally, other skew quadrupole circuits, present in the original design but not used early on, have become operational and also are used routinely in the tuning of the Tevatron. While any or all of these circuits can easily affect the "minimum tune split" in the Tevatron, the strengths required – due to the large a_1 – have a measurable effect on the vertical dispersion function of the Tevatron.

2 Dispersion Measurements

Soon after realization that the uncorrected transverse coupling in the Tevatron was quite strong[2], it was suggested to look at the magnitude of the vertical dispersion in the Tevatron as this should be easily predicted from the inherent coupling and its distributed correction. Dispersion, the variation of the closed orbit with momentum, is a straightforward measurement to make operationally by varying the radial position feedback loop of the RF system. An example of such a measurement is shown in Figure 1. The horizontal positions vary according to the design dispersion in the ring, which has peak values of about $D_x = \Delta x/(\Delta p/p) = 5$ m. However, the vertical dispersion, zero by design, takes on values with peaks of $D_y \approx$ 0.6-0.8 m. A common feature of these recent vertical dispersion measurements has been the coherent oscillatory pattern with frequency near the betatron tune, and with smaller vertical dispersion seen through B- and C-sectors, and much smaller dispersion through A-sector.

With the newly understood strong coupling in the Tevatron, it is straightforward to predict the effects on vertical dispersion given the sources of coupling, including the correction elements. First, we examine the arrangement of skew quadrupole correctors in the Tevatron.

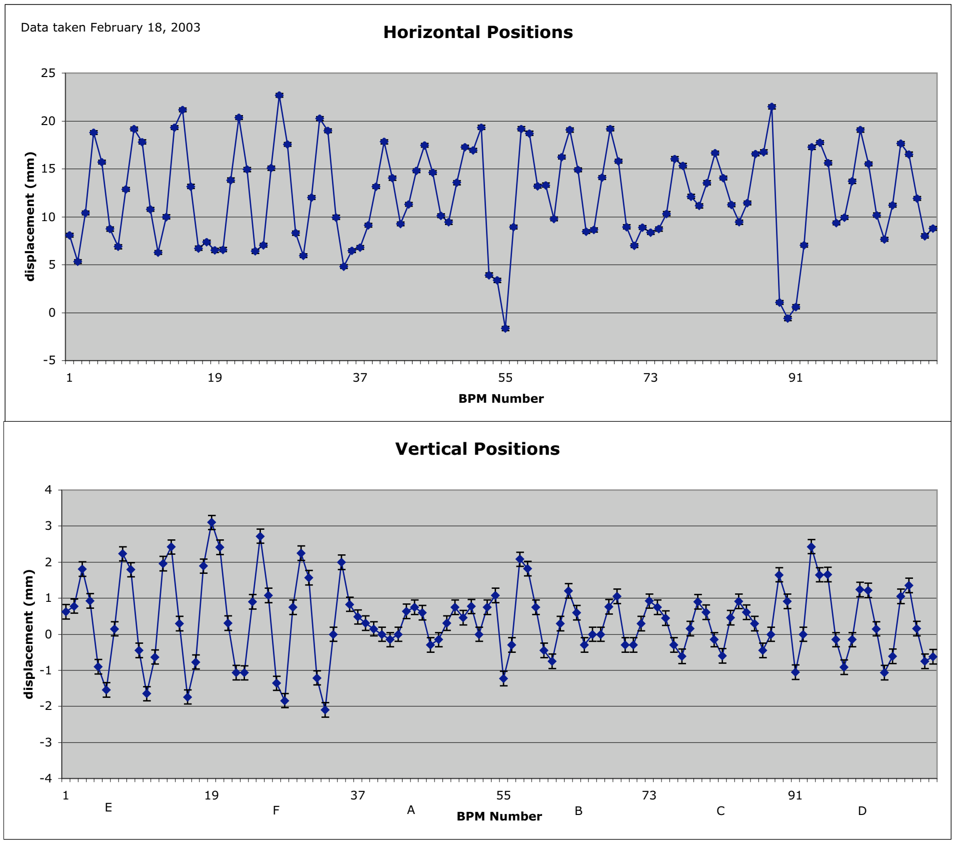

Figure 1: Beam position data from the Tevatron showing the difference between two orbits taken with two different average momenta. Top plot is horizontal positions (±10 mm), bottom plot is vertical positions (±2 mm). The beam position monitors are labeled here starting from E0. (Data taken February 18, 2003, courtesy B. Hanna, G. Annala.)

3 Skew Quadrupole Circuits

Influenced by the Main Ring experience, skew quadrupoles were provided in the Tevatron to handle the $\nu_x - \nu_y = 0$ coupling resonance. The original circuit, dubbed T:SQ in the control system, contained 48 skew quadrupoles, spaced 8 per sector, every other cell throughout the arcs. Additionally, skew quadrupoles on either end of each of the 6 long straight sections were also installed in the Tevatron, though they were not used at the time of Tevatron commissioning.

To make improvements to the Low-Beta optics for Run II collider operations, certain spool pieces were replaced in order to include stronger corrector quadrupoles. The corrector packages in those spool pieces, namely at the 46, 13, and 17 locations near B0 and D0, no longer have T:SQ elements. Also, the skew quadrupoles at the A0, B0, D0, and E0 straight sections have become available for use in Tevatron operations. The B0 skew quads have separate supplies (T:SQA4, T:SQB1), while the other straight sections have their upstream and downstream skew quads on single supplies (T:SQA0, T:SQD0, T:SQE0). Table 1 shows the skew quadrupole configuration available in the Tevatron today.

	A	B	C	D	E	F
	11(A0)	11(B1)		11(D0)	11(E0)	
—	13	○	13	○	13	13
|	17	○	17	○	17	17
|	22	22	22	22	22	22
orig.	26	26	26	26	26	26
T:SQ	32	32	32	32	32	32
|	36	36	36	36	36	36
|	42	42	42	42	42	42
—	○	46	○	46	46	46
	49(A4)		49(D0)	49(E0)		49(A0)

Table 1: Table of skew quadrupole locations (station numbers) in the Tevatron. A "○" indicates locations of missing T:SQ elements.

4 Tuning Procedures

The skew quadrupole circuits have traditionally been used to tune the "global coupling" by monitoring the difference between the horizontal and vertical tunes and adjusting the two "tune quad" circuits and skew quad circuit(s) in order to bring the two eigentunes as close together as possible, typically to the order of $\Delta\nu \equiv |\nu_2 - \nu_1| \approx 0.003$ or better. Due to a single skew quadrupole of strength $k \equiv B'\ell/B\rho$, the minimum tune difference that can be obtained would be $\Delta\nu_{min} = (k/2\pi)\sqrt{\beta_x\beta_y}$. Because T:SQ has 42 (previously 48) elements, at 150 GeV a change of $\Delta\nu = 0.001$ corresponds to a change in T:SQ current of 0.01 A, out of its typical 150 GeV setting of -3 A. The maximum current available from the power supply is ± 50 A. Because the other circuits have only one or two elements, they are often used for fine adjustments of $\Delta\nu$.

The T:SQ circuit has been tuned primarily to compensate the systematic a_1 in the dipole magnets. For $a_1 = +1.4 \times 10^{-4}$/in, the resulting current would be

$$I = -2\pi a_1 I_0 (B\rho/B_0'\ell)(1/42) = -2\pi(1.4 \times 10^{-4}/0.0254)(50\text{A})(500/7.5)(1/42) = -2.7 \text{ A} \qquad (1)$$

which is consistent with its present setting. Other skew quadrupole circuits have been tuned for increased performance day-to-day. When the measurements of Figure 1 were taken, this circuit and the other skew quadrupole circuits had the values shown in Table 2.

It should be pointed out that at 980 GeV the T:SQ circuit runs at -28 A, or 60% of its range. In addition, this is 50% stronger than the simple scaling of its 150 GeV current to 980 GeV.

Circuit	SQ	SQA0	SQA4	SQB1	SQD0	SQE0
Current (A)	-3.0	6.27	-5.17	0.56	0	0

Table 2: Table of currents in the skew quadrupole circuits in the Tevatron on February 18, 2003, corresponding to the data of Figure 1, taken at 150 GeV.

5 Vertical Dispersion due to Coupling

A horizontal orbit offset through a skew quadrupole will generate a deflection in the vertical plane through an angle $\Delta\theta_y = kx$, where $k = B'\ell/B\rho$ is the skew quadrupole strength. For a main quadrupole with focal length F rotated about its axis by a small angle ϕ, $k = 2\phi/F$. For a main dipole magnet which bends the design trajectory through an angle θ_0 and which has a skew quadrupole moment a_1, $k = \theta_0 a_1$. So, if a horizontal orbit offset through a skew quadrupole is generated due to a change in momentum then a vertical orbit distortion will ensue for that momentum. Thus, the skew quadrupole field will generate a vertical dispersion function describing the vertical orbit distortions due to momentum variations. The vertical dispersion due to a single skew quadrupole will be

$$\Delta D_y(s) = \frac{kD_x\sqrt{\beta_k\beta_y(s)}}{2\sin\pi\nu}\ \cos(|\Delta\psi(s)| - \pi\nu) \tag{2}$$

where D_x is the horizontal dispersion function at the location of the quad, β_k is the vertical amplitude function at the location of the quad, ν is the vertical tune, and $\Delta\psi$ is the phase advance from the location of the error.

For random quadrupole rolls in the Tevatron, the expected rms vertical dispersion would be of the order

$$\Delta D_y^{rms} \approx \frac{\phi_{rms}}{F}\frac{\sqrt{\langle D_x^2\beta\rangle\beta_0}}{|\sin\pi\nu|}\sqrt{\frac{N}{2}} \tag{3}$$

for N quadrupoles around the ring. For a FODO system, averaged over the quadrupole locations,

$$\langle D_x^2\beta\rangle = \theta^2 L^3\left(\frac{1-\frac{3}{4}\sin^2\frac{\mu}{2}}{\cos\frac{\mu}{2}\sin^5\frac{\mu}{2}}\right)$$

where θ is the total bend angle within a half-cell of length L and μ is the cell phase advance. For Tevatron parameters, this has a value of about 400 m^3. Thus for 200 quadrupoles, and a β_0 of about 100 m at the BPM locations, we expect a random fluctuation of vertical dispersion with $\Delta D_y^{rms} \approx 8$ cm for an rms roll angle $\phi_{rms} \approx 1$ mrad.

The systematic a_1 and its correction also generate vertical dispersion. For an estimate of the order of magnitude, we look at a standard FODO cell in the Tevatron arc. Each half-cell has 4 dipole magnets and we can assume a kick in the vertical orbit due to energy offset by an amount

$$\Delta\theta_y = 4\theta_0 a_1\bar{D}_x\frac{\Delta p}{p}$$

where $\bar{D}_x$ is the periodic horizontal dispersion at the center of a standard half-cell. The periodic (closed) orbit generated through a repeated series of cells measured at the vertically focusing quadrupoles would be

$$\Delta y = D_y\frac{\Delta p}{p} = \frac{L\Delta\theta_y}{\sin^2(\mu/2)}\left(1+\frac{1}{2}\sin(\mu/2)\right)$$

where L is the half-cell length (30 m for the Tevatron). So we would get for the maximum vertical dispersion occurring at the vertically focusing quadrupole magnets

$$\hat{D}_y = \hat{D}_x\cdot(\bar{D}_x a_1) = (4\text{ m})(3\text{ m }\times 1.4\times 10^{-4}/\text{in}) = 6.6\text{ cm}.$$

The fact that the Tevatron has straight sections which are not dispersion-matched to the cells introduces a dispersion wave on the order of 50%, so that the effect of the systematic a_1 would produce maxima in the vertical dispersion of order 10 cm.

The T:SQ quadrupole correctors, however, are spaced 2 cells apart, and hence typically are correcting locally for errors found in 16 dipole magnets. If we consider a repetitive pattern of one corrector every 2 cells, and note the correctors are located at positions of minimum vertical β, then the periodic dispersion generated by these correctors will be

$$\tilde{D}_y = \frac{(-q\hat{D}_x)\sqrt{\beta_y\breve{\beta}}}{2\sin[2\mu/2]}\cos[|\Delta\psi| - 2\mu/2]$$

which, when operating at -3 A, is a periodic wave of amplitude 7 cm at the vertical BPM locations. Since the correction is of the opposite sign of the errors and the two amplitudes are similar, the net vertical dispersion due to the a_1 skew field and its local correction is roughly zero at the BPM locations, to the level of about a centimeter or so.

From the discussion above, one might expect vertical dispersion in the Tevatron to be mostly a random pattern with an rms on the scale of a few centimeters. However, there remain two important details. First, as can be seen from Table 2 the T:SQ circuit has "missing correctors" at locations near the Interaction Regions. Secondly, additional skew quadrupole circuits are being used to tune the coupling, especially the T:SQA0 and T:SQA4 circuits which are running at currents stronger than T:SQ. (Note, the individual magnets being powered by these circuits are all identical.) This situation will account for the observed vertical dispersion, as we shall see next.

6 Vertical Dispersion in the Tevatron

Spurred by the need for strong coupling correction during slow extraction from the Main Ring, the Tevatron was designed with a skew quadrupole correction circuit using 8 equally spaced magnets per sector which could be used to correct for a systematic skew quadrupole moment in the Tevatron dipole magnets. Since this was such an issue with the commissioning of the Main Ring, the skew quadrupole moment, $a_1 \equiv (\partial B_x/\partial x)/B_0$, of the Tevatron magnets was carefully controlled during construction. After the initial running of the Tevatron, however, movement of mechanical components within the magnets has construed to generate a highly systematic a_1 component, leading to strong coupling in the Tevatron.

Tuning of the usual figure of merit, $\Delta\nu_{min}$, is a reasonable approach to setting the skew quadrupole current in the T:SQ circuit. However, as was described in Section 3, there are missing correctors in the vicinity of both B0 (CDF) and D0 in the Tevatron. There are two issues that this situation presents. Firstly, the vertical dispersion generated by the dipole magnets (*via* a_1) is not cancelled by the correction circuit. Thus, the "missing correctors" represent sources of vertical dispersion. The dispersion generated by one missing corrector would be of approximate magnitude

$$\Delta D_y = \frac{(-kD_x)\sqrt{\beta_k\beta_0}}{2|\sin\pi\nu|} = \frac{(16\theta_0 a_1 D_x)\sqrt{\hat{\beta}\breve{\beta}}}{2|\sin\pi\nu|} = \frac{16\theta_0 a_1 D_x F}{|\sin\pi\nu|} \approx 7.5 \text{ cm}.$$

Surrounding one IR, the three missing correctors are spaced in phase by about 210° and 140°, yielding a dispersion wave outside of the region with a magnitude of about 10 cm. The circumstance that B0 and D0 are almost exactly in phase with each other (the betatron phase advance is approximately $7 \times 2\pi$ between detectors) plus the fact that closed orbit distortions are odd functions of the phase advance implies that the distortions from both regions will add destructively between B0 and D0 and add coherently outside this region. The result is nearly zero vertical dispersion between B0 and D0, and a wave of about 20 cm of vertical dispersion from D0 back around to B0.

However, an even larger dispersion is observed due to a second issue generated by the missing correctors. Since there is one missing corrector upstream of each IR and two missing correctors downstream of each IR, then the center of gravity of the disturbance is not centered at the middle of the long straight sections. In fact, it is centered in the middle of the "13" half-cell. Since the horizontal and vertical phases advance differently through a half-cell, the T:SQ circuit alone will not be able to correct the coupling resonance (or minimum tune split). As it turns out, the circuit with the best (though, not ideal) phase is the T:A0 circuit.[3]. This circuit is 37° out of phase with the T:SQ circuit. Therefore, in order to "decouple" the Tevatron by minimizing the eigentunes' separation, the T:A0 circuit is used and must run at a high current (∼6 A compared to −3 A for T:SQ). The two skew quadrupoles in this circuit are at locations of moderate horizontal dispersion and therefore generate vertical dispersion. While the A0 circuit consists of only 2 elements compared to the 42 elements of the large circuit, the required 6 A to fine-tune the coupling will generate a dispersion wave of amplitude 40 cm. Additionally, the A0 wave will be in phase with the D0-to-B0 wave (due to the missing correctors), and out of phase between A0 and B0.

Three other possible skew quadrupole circuits – T:A4, T:B1, and T:D0 – are at locations of very small horizontal dispersion, but also have very small phase angles with respect to T:SQ. The E0 correctors are at a phase of about 23° compared to T:SQ and so are the next best set to use after T:A0 for decoupling the tunes. However, the horizontal dispersion at E0 is about 2 m and therefore these correctors also will alter the vertical dispersion when they are used.

In conclusion, the vertical dispersion measured in the Tevatron can be thought of primarily as the superposition of orbit distortions from three sources. The first two sources are the two regions of missing skew quadrupole correctors from the original T:SQ circuit, in each of the two interaction regions. These produce a 20 cm wave from D0 clockwise around the ring to B0, and nearly zero dispersion from B0-to-D0. The third source is the strong powering of the T:A0 circuit, which produces a 40 cm wave in phase with the other two clockwise from D0 to A0, and out of phase from A0 to B0. Therefore, the resulting wave should look something like:

- Between A0 and B0: 0.4 - 0.2 = 0.2 m wave
- Between B0 and D0: 0.4 m wave
- Between D0 and A0: 0.4 + 0.2 = 0.6 m wave

A simple calculation, using the design lattice parameters and the corrector circuit settings given in Table 2, gives a rough prediction for the vertical dispersion and is shown in Figure 2. As can be seen, this gives a very fair resemblance to the vertical position data shown in Figure 1.

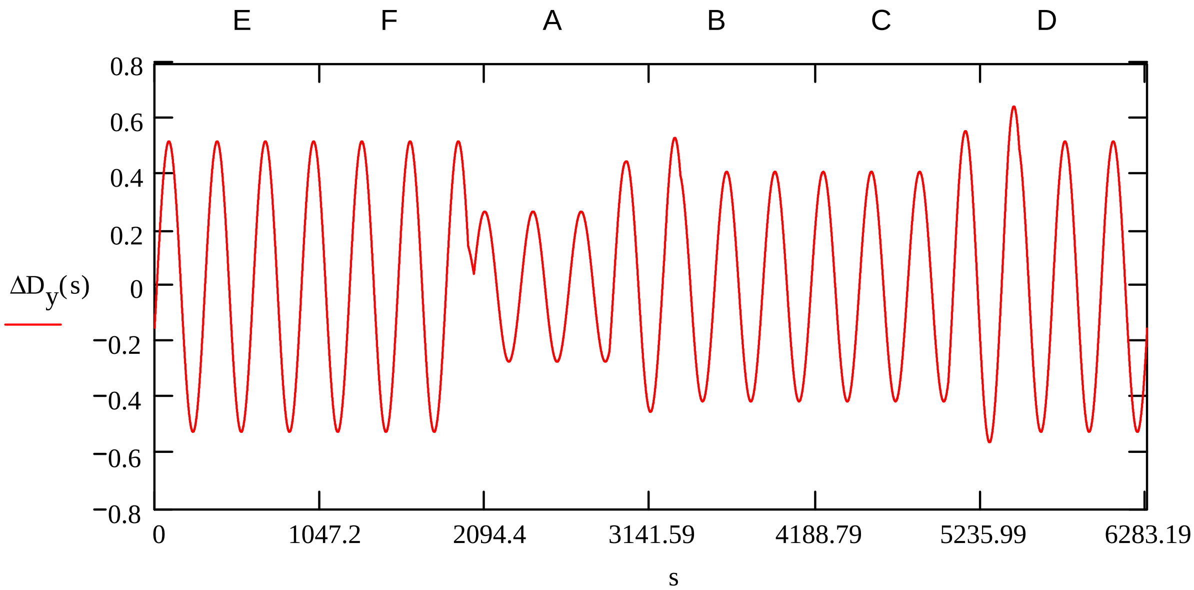

Figure 2: Dispersion pattern generated by currents shown in Table 2 and from "missing" T:SQ components in the interaction regions. The horizontal axis is distance, in meters, from E0. The vertical axis is vertical dispersion, in meters.

7 Skew Quadrupole Tuning and Vertical Disperson

The vertical dispersion function in the Tevatron can change day-to-day due to the tuning of the minimum tune split, $\Delta\nu$, using a variety of skew quadrupole circuits. Figure 3 shows three possible configurations. The top plot shows the disperison generated by the arrangement of corrector currents as was found at the time the data of Figure 1 were taken. Since that time, the E0 circuit has become part of the operation in an attempt to locally decouple the ring near the transverse damper pickups and kickers. The currents more closely resembling today's operation, and the resulting vertical dispersion are seen in the middle plot. The bottom plot shows an attempt to produce zero vertical dispersion through the injection straight section of the Tevatron, F0.

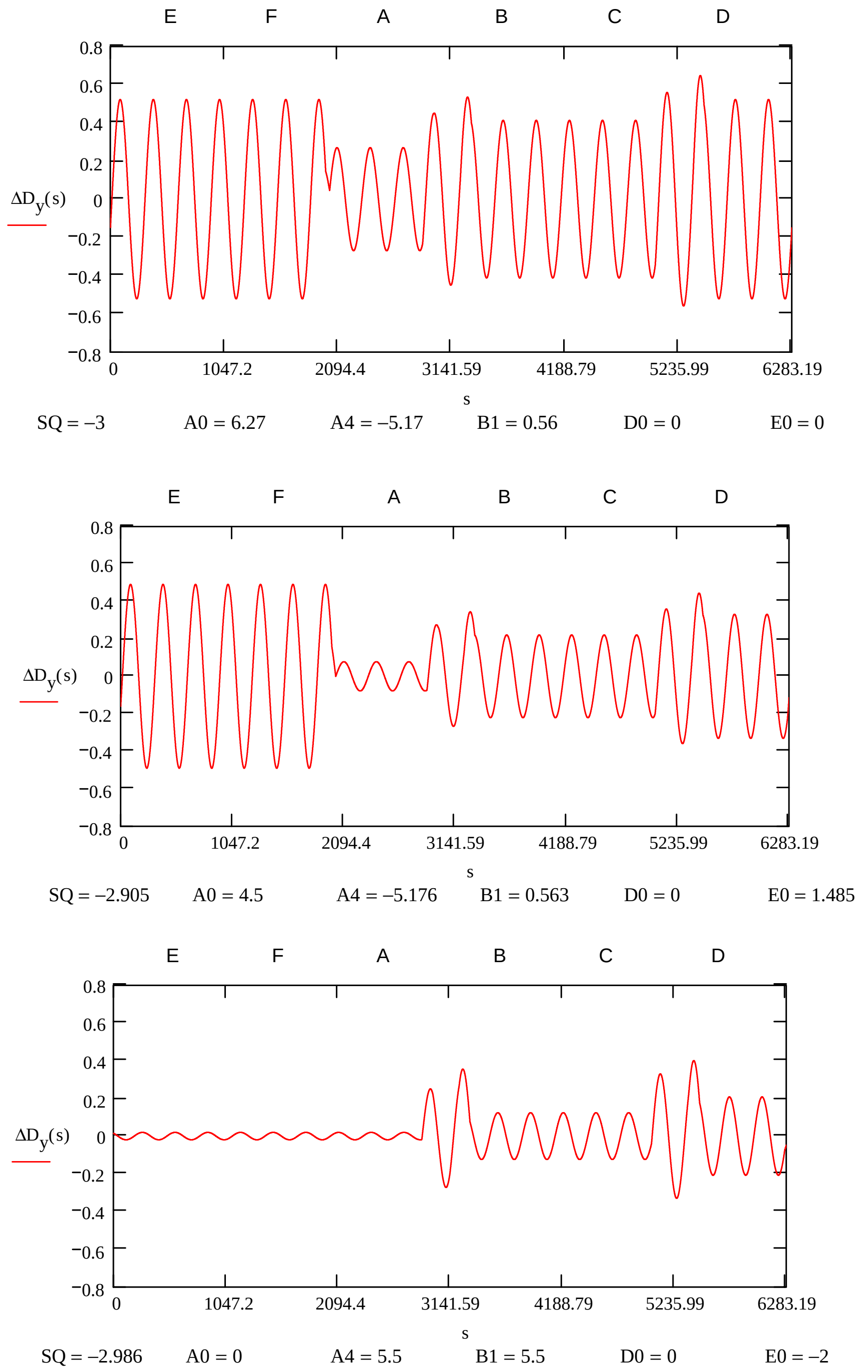

Figure 3: Dispersion generated by various arrangements of skew quadrupole currents. The top figure corresponds to the data taken February 18, 2003; the middle figure corresponds to operating conditions of May 2003. The bottom figure corresponds to an attempt to reduce the vertical dispersion at the injection point (F0). Below each figure are the skew quadrupole circuit settings, in Amperes.

8 Concluding Remarks

This paper shows how the vertical dispersion in the Tevatron can be strongly affected by the tuning of the various skew quadrupole circuits available. Until recently, the only parameter monitored while adjusting these circuits was the minimum attainable difference between the two eigentunes of the transverse motion. An attempt was made to attain the vertical dispersion of the bottom plot in Figure 3.[1] The vertical dispersion was observed indeed to change in the predicted way, and the result had a noticeable effect on the vertical emittance after injection. (The settings of the skew circuits could not be left in this configuration without considerable retuning of other machine parameters, which would have taken far more than the available time alloted for the experiment.)

We have also seen that the greatest source of vertical dispersion in the Tevatron is from the T:A0 circuit, which is running strong due to the missing SQ magnets within the modified interaction regions. The interplay with the A0 dispersion wave with the dispersion left over from these same uncorrected regions of coupling sources generates 0.6 m of dispersion. The random sources of coupling from misaligned main quadrupoles generate another 0.1-0.2 m on top of this, creating peaks in the vertical dispersion of 0.8 m or so.

One peak in the vertical dispersion tends to occur at the F0 injection straight section, and may be responsible in part for the observed vertical emittance dilution during the injection process if the incoming beamline is not properly matched to this value. For reference, the emittance growth due to a dispersion mismatch of amplitude $\Delta\hat{D}_y$ between the incoming beamline and the synchrotron will generate growth in vertical emittance of amount

$$\Delta\epsilon_y \approx 3\pi \frac{(\Delta\hat{D}_y)^2}{\beta_y} \left(\frac{\sigma_p}{p}\right)^2 (\gamma\beta)$$

$= 5\pi$ mm-mrad for $\Delta\hat{D}_y = 1$ m, $\sigma_p/p = 10^{-3}$ at 150 GeV.

References

[1] M. Syphers, "Estimate of Skew Quadrupole FIeld in Tevatron Dipoles due to Creep," Beams-doc-513, March 2003.

[2] D. Edwards and M. Syphers, "StrongTransverse Coupling in the Tevatron," Beams-doc-501, March 2003.

[3] M. Martens, "Notes on Skew Quadrupole Fields in the Tevatron," Beams-doc-485, March 2003.

[1] Data taken May 20, 2003, by G. Annala, D. Edwards, and M. Syphers, to be reported in a separate note.